

Marc Sabat

**Euler Lattice Spirals
Scenery**

for string quartet

PLAINSOUND MUSIC EDITION

Euler Lattice Spirals Scenery (2011/12)

for string quartet

*written for and premiered by the Sonar Quartett Berlin
(Wojciech Garbowski, Cosima Gerhardt, Nikolaus Schlierf, Susanne Zapf)*

*composed during a one-year residency in Rome
at the Accademia Tedesca Villa Massimo*

This work is the third in an ongoing cycle of string quartets in which musical forms emerge as consequences of explicitly notated intonation. The title refers to Leonhard Euler's discovery of a two-dimensional diagram representing the harmonic space subset based on octave equivalence, pure fifths and thirds: a tonal lattice that models triadic harmony, or in James Tenney's terminology: *(3,5) projection space*.

An extended portion of this lattice, comprising 99 distinct microtonal pitch-classes organised as a progression of major and minor triads tuned in Just Intonation, forms the basis of the fourth movement, *Harmonium for Ben Johnston*. Each triad occurs only once, and for the most part connects to its neighbors by a shared common tone, until reaching the small enharmonic seam in the middle of the movement, from which point a retrograde inversion of the triads begins. The triads are ordered in such a way that all possible common-tone progressions are explored, and also that the progression of triads which opens the piece recurs in the middle of the movement, transposed upward by two commas.

To realize this modulation into distant regions, the open strings of the quartet must be precisely tuned in 3:2 ratio untempered fifths, so that the comma distinctions and partial unisons between open strings may be optimised. Thus, the composition is completed by four additional movements, which explicitly compose the tuning procedure (*Preludio*), investigate the Pythagorean sonorities of the lower natural harmonics (*Pythagoras Drawing I and II*), and present the unisons and commas of higher natural harmonics in the registers they occur (*Harmonium for Claude Vivier*) as an ecstatic singing melody.

Rome, 28 October 2011 / Berlin, 6 March 2012

An informal introduction to the Helmholtz-Ellis Accidentals

by Marc Sabat

Berlin, April 2009

In learning to read HE accidentals, without having to rely on an electronic tuning device, it is important to be familiar with three things:

First, to keep in mind the natural tuning of intervals in a harmonic series, which deviate from the tempered system.

Second, to get to know how the accidentals refer to these overtone relationships.

Third, to observe that each written pitch may be related to many other pitches by natural intervals, and to tune it accordingly.

In most cases, this approach will allow the player to quickly and intuitively play just intonation (JI) pitches quite accurately. Any remaining adjustments can be made by ear, based on the specific sound of JI intervals.

Just intervals are readily learned because they are built up from simple, tuneable harmonic relationships. These are generally based on eliminating beating between common partials, finding common fundamentals and audible combination tones, and establishing a resonant, stable sonority which maximizes clarity: both of consonance and of dissonance.

A well-focussed JI sound is completely distinct from the irregular, fuzzy beating of tempered sounds. Just consonances, when marginally out of tune, beat slowly and sweetly and may be corrected with the most subtle adjustments of bowing or breath. Just dissonances produce a sharply pulsing regular rhythm and have very clear, distinct colors.

To become familiar with the notation and sounds of JI, the fundamental building blocks are prime number overtones 3, 5, 7, 11 and 13, each of which is associated with a specific pair of accidentals and a basic musical interval.

3 is associated with the signs flat, natural, sharp and refers to the series of untempered perfect fifths (Pythagorean intonation). Generally, A is taken as the tuning reference, and the central pitches C-G-D-A-E can be imagined as the normal tuning of the orchestral string instruments. The just C is rather lower than tempered tuning because of the pure fifths. The further this series is extended, the greater the deviation from tempered tuning: the flats are lower, the sharps higher.

5 is associated with arrows attached to the flat, natural, sharp signs and refers to the pure major third. These arrows correct the Pythagorean intervals by a Syntonic Comma, which is approximately $\frac{1}{9}$ of a whole tone or 22 cents. So, for example, the note E-flat arrow-up is a just major third below G, and the note F-sharp arrow-down is a major third above D. In most music, flats are often raised by a comma and sharps are lowered. Because of the open string tuning, it is common to sometimes raise F and C (to match A and E) and to sometimes lower A and E (to match F and C). Corrections by one Syntonic Comma have been used throughout Western music history and are relatively familiar to the ear. However, traditionally these corrections have been hidden by players, for example in Meantone Temperament where fifths are mistuned narrow by $\frac{1}{4}$ comma so that the third C-E ends up sounding pure. More recently, the currently prevailing Equal Temperament has made us accustomed to beating thirds, so at first the pure intervals may seem unfamiliar. To play the arrows accurately, one must carefully learn the sound of the consonant major and minor thirds and sixths, and learn to articulate comma differences clearly.

7 is associated with a Tartini sign resembling the numeral. It corrects the Pythagorean intervals by a Septimal Comma, which is approximately $\frac{1}{7}$ of a whole tone or 27 cents. When the Pythagorean minor third is lowered by this amount, it becomes a noticeably low third often heard in Blues music.

11 is associated with the quartertone signs (cross and backwards flat). The accidental is used to raise the perfect fourth by 53 cents, producing the exact tuning of the 11th partial in a harmonic series. The sound is most easily learned by playing one octave plus one fourth and raising it by a quartertone.

13 is associated with the thirddtone signs (cross and backwards flat, each with 2 verticals). The accidental is used to lower the Pythagorean major sixth by 65 cents, producing the exact tuning of the 13th partial in a harmonic series. The sound is most easily learned as a neutral-sounding sixth, one-third of the way between the just minor and just major sixths (closer to minor than to major).

The following table presents the accidentals together with their associated ratios and cents deviations. To calculate the cents deviation from Equal Temperament of a specific written pitch (if desired) the following shortcut may be used:

- 1.) Find the cents deviation of the Pythagorean pitch, by calculating how many fifths it is away from A, multiplying by 2, and using a plus sign if it is on the sharp side and a minus if it is on the flat side.
- 2.) For each microtonal accidental, add or subtract its approximate cents value (as given above), keeping in mind whether the accidental is raising or lowering the pitch.

The resulting value should be a cents deviation within 1 or 2 cents accuracy, which is an acceptable starting point for fine-tuning by ear.

ACCIDENTALS

EXTENDED HELMHOLTZ-ELLIS JI PITCH NOTATION

for Just Intonation

designed by Marc Sabat and Wolfgang von Schweinitz

The exact intonation of each pitch may be written out by means of the following harmonically-defined signs:

$\flat\flat$ \flat \natural \sharp \times *Pythagorean series of fifths – the open strings*
(... c g d a e ...)

\flat \natural \sharp \times $\flat\flat$ \flat \natural \sharp
lowers / raises by a syntonic comma
 $81 : 80 = \text{circa } 21.5 \text{ cents}$

\flat \natural \sharp \times $\flat\flat$ \flat \natural \sharp
lowers / raises by two syntonic commas
circa 43 cents

\lrcorner \llcorner
lowers / raises by a septimal comma
 $64 : 63 = \text{circa } 27.3 \text{ cents}$

\llcorner \lrcorner
lowers / raises by two septimal commas
circa 54.5 cents

\dagger \dagger
raises / lowers by an 11-limit undecimal quarter-tone
 $33 : 32 = \text{circa } 53.3 \text{ cents}$

$\#$ $\#$
lowers / raises by a 13-limit tridecimal third-tone
 $27 : 26 = \text{circa } 65.3 \text{ cents}$

\approx \approx
lowers / raises by a 17-limit schisma
 $256 : 255 = \text{circa } 6.8 \text{ cents}$

\nearrow \searrow
raises / lowers by a 19-limit schisma
 $513 : 512 = \text{circa } 3.4 \text{ cents}$

\uparrow \downarrow
raises / lowers by a 23-limit comma
 $736 : 729 = \text{circa } 16.5 \text{ cents}$

In addition to the harmonic definition of a pitch by means of its accidentals, it is also possible to indicate its absolute pitch-height as a cents-deviation from the respectively indicated chromatic pitch in the 12-tone system of Equal Temperament.

The attached arrows for alteration by a syntonic comma are transcriptions of the notation that Hermann von Helmholtz used in his book “Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik” (1863). The annotated English translation “On the Sensations of Tone as a Physiological Basis for the Theory of Music” (1875/1885) is by Alexander J. Ellis, who refined the definition of pitch within the 12-tone system of Equal Temperament by introducing a division of the octave into 1200 cents. The sign for a septimal comma was devised by Giuseppe Tartini (1692-1770) – the composer, violinist and researcher who first studied the production of difference tones by means of double stops.

VORZEICHEN

EXTENDED HELMHOLTZ-ELLIS JI PITCH NOTATION

für die natürliche Stimmung

konzipiert von Marc Sabat und Wolfgang von Schweinitz

Die Stimmung jedes Tons ist mit folgenden harmonisch definierten Vorzeichen ausnotiert:

$\flat\flat$ \flat \natural \sharp \times

Pythagoreische Quintenreihe der leeren Streicher-Saiten
(... c g d a e ...)

\flat \natural \sharp \times $\flat\flat$ \flat \natural \sharp

Erniedrigung / Erhöhung um ein Syntonisches Terzkomma
 $81 : 80 = \text{circa } 21.5 \text{ cents}$

\flat \natural \sharp \times $\flat\flat$ \flat \natural \sharp

Erniedrigung / Erhöhung um zwei Syntonische Terzkommas
 $\text{circa } 43 \text{ cents}$

\lrcorner \llcorner

Erniedrigung / Erhöhung um ein Septimenkomma
 $64 : 63 = \text{circa } 27.3 \text{ cents}$

\llcorner \lrcorner

Erniedrigung / Erhöhung um zwei Septimenkommas
 $\text{circa } 54.5 \text{ cents}$

\dagger \dagger

Erhöhung / Erniedrigung um den undezimalen Viertelton der 11er-Relation
 $33 : 32 = \text{circa } 53.3 \text{ cents}$

\mathbb{H} \mathbb{H}

Erniedrigung / Erhöhung um den tridezimalen Drittelton der 13er-Relation
 $27 : 26 = \text{circa } 65.3 \text{ cents}$

\approx \approx

Erniedrigung / Erhöhung um ein Siebzehner-Schisma
 $256 : 255 = \text{circa } 6.8 \text{ cents}$

\nearrow \searrow

Erhöhung / Erniedrigung um ein Neunzehner-Schisma
 $513 : 512 = \text{circa } 3.4 \text{ cents}$

\uparrow \downarrow

Erhöhung / Erniedrigung um ein Dreiundzwanziger-Komma
 $736 : 729 = \text{circa } 16.5 \text{ cents}$

Zusätzlich zu der harmonischen Definition der Tonhöhe durch das Vorzeichen für jeden Ton ist auch der Cents-Wert der Abweichung der gewünschten Stimmung von der Tonhöhe des jeweils bezeichneten chromatischen Tons der gleichstufig temperierten Zwölfton-Skala angegeben.

Die attachierten Pfeile für die Alteration um ein Syntonisches Terzkomma sind eine bloße Transkription der Notation, die Hermann von Helmholtz in seinem Buch "Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik" (1863) verwendet hat. Die kommentierte englische Übersetzung "On the Sensations of Tone as a Physiological Basis for the Theory of Music" (1875/1885) stammt von Alexander J. Ellis, der auch eine enorme Verfeinerung der Tonhöhendefinition innerhalb des Zwölftonsystems der gleichstufig temperierten Stimmung durch die Unterteilung der Oktave in 1200 Cents eingeführt hat. – Das Vorzeichen für die Alteration um ein Septimenkomma wurde von Giuseppe Tartini (1692-1770) erfunden, der als Komponist, Geiger und Wissenschaftler die durch Doppelgriffe erzeugten Differenztöne untersucht hat.

Euler Lattice Spirals Scenery

Preludio : Les Quintes Justes

Marc Sabat

Tempo ad libitum, take time necessary to sound a precise, stable intonation *
Repeat and move through the patterns freely, going back if and when needed

Violin I

Violin II

Viola

Violoncello

mezza voce, tenuto

sotto voce

mezza voce (m.v.)

sotto voce (s.v.)

m.v.

mezza voce, tenuto

sotto voce <> (s.v.), libero

ppp molto preciso

mezza voce (m.v.)

s.v.

m.v.

mezza voce (m.v.)

s.v. libero

mezza voce (m.v.)

sotto voce <> (s.v.), libero

s.v. <>

libero

* in the course of this section, the open strings are to be tuned just; measured in cents relative to 12-tone equal temperament : E +2, A 0, D -2, G -4, C -6 before playing, it is sufficient to have the strings relatively close to a good tuning, with the cello A-string tuned exactly (preferably to 220 Hz) — the written patterns may be interwoven dynamically and repeated freely in the manner of normal tuning procedure, to achieve the most precise tuning as swiftly as possible; the sound of actually tuning the strings by adjusting the pegs is intentionally part of the music; accuracy may be facilitated by using fine tuners for each string

8

2

s.v. <>

2'/II, 2'/III, 3'/III, II, III, II

even, clear bowing

III

ppp molto preciso

ppp molto preciso

m.v.

H

Detailed description: This system contains measures 8 and 9. The first staff (treble clef) has a measure rest in measure 8 and begins measure 9 with a note marked '2'. The second staff (treble clef) has a measure rest in measure 8 and begins measure 9 with a note marked 's.v.' and '<>'. The third staff (bass clef) has a measure rest in measure 8 and begins measure 9 with a note marked 'III'. The fourth staff (bass clef) has a measure rest in measure 8 and begins measure 9 with a note marked '+14 III'. Dynamic markings include 'ppp molto preciso' and 'm.v.'. Performance instructions include 'even, clear bowing' and 'H'.

10

H

m.v.

-4

-20

p molto preciso

2'/III, +14, IV

ppp molto preciso

m.v.

m.v.

m.v.

even, clear bowing

p molto preciso

s.v. <>

2'/II, 2'/III, 3'/III, II, III, II

Detailed description: This system contains measures 10 and 11. The first staff (treble clef) has a measure rest in measure 10 and begins measure 11 with a note marked 'm.v.'. The second staff (treble clef) has a measure rest in measure 10 and begins measure 11 with a note marked '2'/III'. The third staff (bass clef) has a measure rest in measure 10 and begins measure 11 with a note marked 'm.v.'. The fourth staff (bass clef) has a measure rest in measure 10 and begins measure 11 with a note marked 'm.v.'. Dynamic markings include 'p molto preciso' and 'ppp molto preciso'. Performance instructions include 'even, clear bowing' and 'H'.

12

2'/III, 2'/IV, 3'/IV, III, IV, III

s.v. <>

ppp molto preciso

H

m.v.

-4

-20

p molto preciso

2'/II, 2'/III, 3'/III, II, III, II

m.v.

even, clear bowing

ppp molto preciso

-18

Detailed description: This system contains measures 12 and 13. The first staff (treble clef) has a measure rest in measure 12 and begins measure 13 with a note marked 's.v.' and '<>'. The second staff (treble clef) has a measure rest in measure 12 and begins measure 13 with a note marked 'm.v.'. The third staff (bass clef) has a measure rest in measure 12 and begins measure 13 with a note marked 'm.v.'. The fourth staff (bass clef) has a measure rest in measure 12 and begins measure 13 with a note marked 'm.v.'. Dynamic markings include 'p molto preciso' and 'ppp molto preciso'. Performance instructions include 'even, clear bowing'.

14

3[/]III 2[/]IV

2[/]III 2[/]IV 3[/]IV III IV III

+16 tune to A harmonic in VI1

ppp
molto preciso

m.v.

H

H

m.v.

m.v.

16

III

-18 *ppp*
molto preciso

m.v.

s.v.

-35 *ppp*
molto preciso

m.v.

2[/]III 2[/]IV 3[/]IV III IV III

s.v.

s.v.

2[/]III 2[/]IV 3[/]IV III IV III

s.v.

s.v.

18

I

s.v.

I beating!

s.v.

beating!

5[/]IV

m.v.

I

ppp
molto preciso

m.v.

ppp
molto preciso

attaca

Pythagoras Drawing (I)

Tempo ad libitum, vary from bar to bar as desired, swinging, like a French Overture
ca. 66-99

martelé, full bow

The musical score is written for four staves: Violin I, Violin II, Viola, and Cello/Double Bass. It is in 3/4 time and consists of 14 measures. The score includes various performance instructions and technical markings:

- Violin I:** Starts with a *mezza voce, sostenuto* instruction. Later, it features *ff* dynamics and *sempre simile* markings. Technical markings include *lead*, *martelé, full bow*, and fingerings (I, II, III, IV, 3^{/III}).
- Violin II:** Features *ff* dynamics and *sempre simile* markings. Technical markings include *lead*, *martelé, full bow*, and fingerings (I, II, III, IV).
- Viola:** Features *ff* dynamics and *sempre simile* markings. Technical markings include *lead*, *martelé, full bow*, and fingerings (I, II, III, IV).
- Cello/Double Bass:** Features *ff* dynamics and *sempre simile* markings. Technical markings include *lead*, *martelé, full bow*, and fingerings (I, II, III, IV, 3^{/III}, 4^{/IV}, 6^{/II}). A specific instruction reads: *molto sul tasto silently press down D-string*.

Additional markings include *mezza voce, sostenuto*, *molto sul tasto silently press down D- and G-strings*, and *come prima*. The score concludes with an *attaca* sign.

* grace notes placed at beginning and end of bar are not to be deliberately synchronized between the instruments

Harmonium for Claude Vivier

Joyeux

♩ = 288 (♩ = 144 | ♩ = 96)

(This staff shows the hocketing counterpoint between parts, written at sounding pitch)

The first system of the musical score covers measures 1 through 6. It includes a 'melodic partcell' staff at the top, which shows the hocketing counterpoint between parts. Below this are staves for VI I, VI II, Vla, and Vlc. The Vlc part includes annotations such as *f sonore e sostenuto possibile* and *sonore e sostenuto possibile* with dynamic markings *f*, *p*, and *f*. A note with a diamond head and a small circle is marked with an asterisk (*). Fingering and voicing instructions like *2[/]IV* and *III* are present. A text box provides instructions: (harmonics are mostly written at the nodes as diamond noteheads inflected by a small circle; the desired partial and string are indicated above; please observe the changing nodes where possible; double-stop harmonics generally take two stems.)

The second system of the musical score covers measures 7 through 14. It features the same five staves as the first system. The Vlc part continues with dynamic markings *f*, *p*, and *f*, and includes *sonore e sostenuto possibile* instructions. A note with a diamond head and a small circle is marked with an asterisk (*). Fingering and voicing instructions like *2[/]IV*, *III*, and *II* are present.

The third system of the musical score covers measures 15 through 22. It features the same five staves. The Vlc part includes dynamic markings *f*, *p*, *f*, *p*, *f*, *p*, and *f*, and includes *sonore e sostenuto possibile* instructions. A note with a diamond head and a small circle is marked with an asterisk (*). Fingering and voicing instructions like *III*, *II*, *IV*, *3[/]IV*, *5[/]IV*, and *5[/]IV* are present.

* this section is played on open strings and natural harmonics whenever possible, with the exception of occasional stopped pitches

25

(p) f -20 p -37 p f p

III (p) f p (dissonant fourth 27/20) f -20 p -37 p

III IV (f) p 1 III III -37 f p p

5/IV 7/IV 2/III 7/IV 5/IV III 2/IV 7/IV 5/IV

31

3/IV 3/IV 2/III IV p -18

3/IV IV III 3/IV 2/III IV (p) p

4/IV IV 3/III sim. 5/IV (p) f (f) III

5/III 5/IV III IV III sim. II III (f)

38

IV f p -37 f p

p p -20 p -20 p

III 7/IV I IV IV III IV (f) p f IV

43

IV
f p f p

III IV
p (f) p f p

IV III IV III II
p f (f) III

sim. 4[/]III 5[/]IV 7[/]III
(f)

52

III I 2[/]II IV III IV IV
(p) f (f) (f) p

-18
p p (p) f

II III 5[/]IV II III IV III IV III IV
p f f p f p f

5[/]III 7[/]III 4[/]IV I 7[/]III 8[/]IV 5[/]II 5[/]III 4[/]III 5[/]IV III
(f) p

59

IV III IV III I
p (p) f

4[/]IV 3[/]III IV III II I
p f (p) f

IV III 3[/]II 7[/]IV 2[/]I 5[/]III 5[/]II
p p (p) f

7[/] 8[/] 9[/] 6[/]IV 5[/] 5[/]IV 5[/]III 5[/]II
f p f (f) IV III

64

Violin I: *f*, *p*, *f*, *p*, *p*, *p*, *f*

Violin II: *p*, *f*, *p*, *f*, *p*, *f*, *f*

Viola: *f*, *p*, *f*, *p*, *f*, *p*, *f*

Cello/Double Bass: *p*, *f*, *p*, *f*, *p*, *f*, *f*

Bass: *p*, *f*, *p*, *f*, *p*, *f*, *f*

71

Violin I: *f*, *p*, *f*, *f*, *p*

Violin II: *f*, *p*, *f*, *f*, *p*

Viola: *f*, *p*, *f*, *f*, *p*

Cello/Double Bass: *f*, *p*, *f*, *f*, *p*

Bass: *f*, *p*, *f*, *f*, *p*

76

Violin I: *p*, *f*, *f*, *p*, *p*

Violin II: *p*, *f*, *f*, *p*, *p*

Viola: *p*, *f*, *f*, *p*, *p*

Cello/Double Bass: *p*, *f*, *f*, *p*, *p*

Bass: *f*, *p*, *f*, *f*, *p*

82

Musical score for measures 82-88. The score is in 2/4 time and consists of five staves. The first staff is the treble clef with a key signature of one flat. The second staff is the treble clef with a key signature of one flat and dynamic markings *f*, *p*, and *(p)*. The third staff is the treble clef with a key signature of one flat and dynamic markings *(p)*, *f*, *p*, *p*, *f*, *p*, and *f*. The fourth staff is the treble clef with a key signature of one flat and dynamic markings *(f)*, *p*, *f*, *p*, *f*, and *p*. The fifth staff is the bass clef with a key signature of one flat and dynamic markings *(f)*. Fingering numbers (I-IV) are indicated throughout the score.

89

Musical score for measures 89-96. The score is in 15/8 time and consists of four staves. The first staff is the treble clef with a key signature of one flat. The second staff is the treble clef with a key signature of one flat and dynamic markings *f* and *p*. The third staff is the treble clef with a key signature of one flat and dynamic markings *(f)*, *p*, *p*, and *f*. The fourth staff is the bass clef with a key signature of one flat and dynamic markings *(f)* and *(f)*. Fingering numbers (I-IV) are indicated throughout the score.

97

Musical score for measures 97-104. The score is in 9/8 time and consists of four staves. The first staff is the treble clef with a key signature of one flat and dynamic markings *p* and *f*. The second staff is the treble clef with a key signature of one flat and dynamic markings *(p)* and *p*. The third staff is the treble clef with a key signature of one flat and dynamic markings *f*, *(f)*, *p*, *f*, and *p*. The fourth staff is the bass clef with a key signature of one flat and dynamic markings *(f)* and *(f)*. Fingering numbers (I-IV) are indicated throughout the score.

Musical score for measures 103-107. The score is written for five staves: Treble Clef 1, Treble Clef 2, Treble Clef 3, Treble Clef 4 (marked with an asterisk and 'artificial harmonic'), and Bass Clef. Measure numbers 103, 104, 105, 106, and 107 are indicated at the beginning of each staff. The music features complex chordal textures with various voicings and articulations. Dynamics include *f* and *p*. Chord symbols such as 7/III, 5/IV, 8/IV, 4/II, 3/I, 5/III, 7/IV, 7/II, 5/II, 8/III, 8/II, 7/I, and 7/IV are present. A specific instruction '* artificial harmonic' is noted above the fourth staff.

Musical score for measures 108-115. The score is written for five staves: Treble Clef 1, Treble Clef 2, Treble Clef 3, Treble Clef 4 (marked with an asterisk and 'artificial harmonic'), and Bass Clef. Measure numbers 108, 109, 110, 111, 112, 113, 114, and 115 are indicated at the beginning of each staff. The music continues with complex chordal textures. Dynamics include *p* and *f*. Chord symbols such as 8/I, 7/II, 5/II, 8/II, 7/II, 7/III, 5/II, 7/IV, 3/II, 5/I, 7/II, 7/I, 7/IV, 5/II, 6/II, and 6/I are present.

Musical score for measures 116-123. The score is written for five staves: Treble Clef 1, Treble Clef 2, Treble Clef 3, Treble Clef 4 (marked with an asterisk and 'artificial harmonic'), and Bass Clef. Measure numbers 116, 117, 118, 119, 120, 121, 122, and 123 are indicated at the beginning of each staff. The music continues with complex chordal textures. Dynamics include *f*, *p*, and *(p)*. Chord symbols such as IV, 7/III, 7/I, 8/I, II, 5/III, 7/IV, 8/III, 5/II, 7/III, 7/II, IV, 7/IV, 5/II, 7/I, 5/IV, and III are present.

124

Violin I: *p*, *f*, *p*, *p*, *f*, *f*, *f*, *f*, *f*, *f*, *f*, *f*

Violin II: *p*, *f*, *p*, *p*, *f*, *f*, *f*, *f*, *f*, *f*, *f*, *f*

Viola: *p*, *(p)*, *(p)*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*

Cello/Double Bass: *p*, *(p)*, *f*, *p*, *p*, *p*, *f*, *f*, *f*, *f*, *f*, *f*

Bass: *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*

Chord diagrams: III, 5^b/IV, III, IV, II, I, IV, II, III, 5^b/III, 7^b/IV, I, II, 6^b/IV, II, 7^b/II, IV, III, 5^b/I, I, II, 7^b/III, 5^b/IV

133

Violin I: *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*

Violin II: *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*

Viola: *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*

Cello/Double Bass: *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*

Bass: *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*

Chord diagrams: III, II, III, IV, I, II, III, IV, 5^b/IV, III, 5^b/III, 5^b/II, I, 7^b/III, 8^b/IV, 7^b, III

137

Violin I: *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*

Violin II: *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*

Viola: *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*

Cello/Double Bass: *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*, *p*

Bass: *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*, *(f)*

Chord diagrams: III, IV, IV, III, IV, II, IV, III, II, I

Harmonium for Ben Johnston

Cantabile

♩ = 72

VI I
mezzo voce
poco f
poco vib. (narrow)
sfz
f

VI II
(arco) c.l.b. → ord.
martelé
sfz > p
f sostenuto

Vla
mezzo voce
f

Vlc
mezzo voce
espr.
poco f

5 +16
f
sfz
p
m.v.

VI I
f
sfz
p
m.v.

VI II
f
sfz
p
m.v.

Vla
m.v.
poco f
p
m.v.

Vlc
martelé
f
m.v.
poco f
p

10
ppp
p
ppp
p
ppp
p
mp

VI I
ppp
p
ppp
p
ppp
p
mp

VI II
ppp
p
ppp
p
ppp
p
mp

Vla
p
poco
ppp
p
ppp
p
mp

Vlc
ppp
p
ppp
p
ppp
p
mp

14

f sfz *poco f* *p* *m.v. espr.* *poco vibr., come prima* *m.v. espr.* *pp*

18

pp *poco vib. (narrow)* *pp* *poco f* *espr.* *ppp* *poco f* *espr.* *pp* *poco f*

22

p *p* *sfz* *p* *l.h. pizz.* *p* *fast vibr.* *p* *sfz* *p*

26

f *p* *poco* *espr.* *sost.* *poco f* *p* *m.v.*

30

30

+27 +43 +27 V +31 *rinf.*

diminuendo

+43 +14 +41 +45 +29 +33 +18

m.v. *diminuendo*

+43 +45 +43 +31 +33 +31

diminuendo

34

34

+20 V +4 +6 +8 +20 *f* *f sost.*

IV

+47 +33 +35 +20

38

38

+35 +49 +35 V +37 +51 +20 +35 +33 +47 +33 +35 +22 +33

p *sfz* *poco f* *m.v.*

42

42

+16 +31 +51 +29 +31 +18 +20 +47 +29 +18 +45 +16 +33

sul tasto *sfz* *pp* *ppp* *poco* *espr.* *l.h. pizz.* *molto sul tasto*

(slow) *V*

46 *ord.*

sfz pp

sfz pp

pp

f sonore

49 *strike the string!*

f

strike the string!

f

p

poco f

fast ricochet

m.v.

p

p

crescendo

m.v.

53 *flautando* *ord.*

m.v.

poco f

sfz

fast ricochet

poco f

f sonore

slow bow

espr.

p

f

57 *p* +23 +37 *p* +23

60 *p* +51 +35 +22

61 *m.v.* +41 +43 *p* +25 +29 +41 +25 +43 *m.v.* +23 +39 +23 +39 +27 *pp* *poco f* *p crescendo*

66 *espr.* +55 +39 +43 *fast ricochet* *poco f* +41 +39 +37 +53 +59 +39 *m.v.* *espr.* (artificial harmonic cents = sounding pitch deviation) +41 II 6.5 III +57 *poco f* *diminuendo*

70 *diminuendo* +61 +31 +43 +61 *port.* *pp sotto voce* +47 +59 +47 +29 +45 +59 +45

(dim.)

Un poco largo

♩ = 57.6

75

G# -39

(dim.)

+4.2 c

-35

libero

* I

-65

-49

p

ppp

change bow as needed

-49

-51

poco f e sempre sostenuto

+47

+4.2 c

+4.2 c

-49

-51

(diminuendo on F only)

(dim.)

sotto voce

77

* IV

libero

-63

p

ppp

in tempo

ppp

in tempo

-63

III

rinf.

-33

libero

in tempo

-33

78

libero II

-49

-35

ppp

libero

-49

-65

-35

ppp

rinf.

5

-35

* l.h.: establish, maintain, fine tune hand positions — hold fingers as long as possible
 r.h.: molto flautando, change bows and vary speed often and irregularly — emerging into sharp focus and receding again
 repeats as many times as needed to set an accurate intonation of the written pattern, sempre un poco ad libitum

simile (tempo libero)

79

poco f
rinf.
m.v.

simile (tempo libero)

mp
pp
poco f
ppp

sfz

80

rinf.
p
sfz

81

molto tasto
ord.
ppp
ppp
ppp

lower E (5 commas down) is dissonant (27/20) with Vln II A!

82

poco f
sfz
rinf.
rinf.

83

ppp

ppp

ppp

84

rinf.

poco f

rinf.

85

rinf.

m.v.

poco f

cantabile, come prima

III

88

rinf.

cantabile, come prima

L'istesso tempo ma giusto, scorrevole

91

poco f

f

m.v. *dolce*

IV

m.v.

m.v.

m.v.

G# tacet ad lib.

94

m.v.

port.

m.v.

6/7 = E -41

98

strike the string!

sfz

* B natural is one schisma (circa 2ε) higher than C flat one comma raised (almost the same pitch)

$\text{♩} = 72$

101

Musical score for measures 101-104. The score is in 3/4 time and consists of four staves: Violin I, Violin II, Cello/Double Bass, and Bass. Measure 101 features a *poco f sostenuto* dynamic. Measure 102 has a *sfz* dynamic. Measure 103 has a *rinf.* dynamic. Measure 104 has a *rinf.* dynamic. Fingerings are indicated with numbers 1-4. Accents and slurs are present throughout.

105

Musical score for measures 105-108. The score is in 3/4 time and consists of four staves: Violin I, Violin II, Cello/Double Bass, and Bass. Measure 105 has a *sfz* dynamic. Measure 106 has a *f sonore* dynamic. Measure 107 has a *sfp espr.* dynamic. Measure 108 has a *poco f* dynamic. Fingerings are indicated with numbers 1-4. Accents and slurs are present throughout.

109

Musical score for measures 109-112. The score is in 3/4 time and consists of four staves: Violin I, Violin II, Cello/Double Bass, and Bass. Measure 109 has a *poco f* dynamic. Measure 110 has a *pp* dynamic. Measure 111 has a *rinf.* dynamic. Measure 112 has a *p* dynamic. A note in measure 111 is marked "one comma lower!". Fingerings are indicated with numbers 1-4. Accents and slurs are present throughout.

113

Musical score for measures 113-115. The score is in 3/4 time and consists of four staves. Measure 113 features a treble clef with a key signature of one sharp (F#) and a 3/4 time signature. It includes a triplet of eighth notes with a finger number of -37 and a slur. The second staff has a V-shaped fingering above a note with a finger number of -25. Measure 114 continues with a V-shaped fingering above a note with a finger number of -39. Measure 115 features a slur over a triplet of eighth notes with a finger number of -23, and a *poco f* dynamic marking. The bass staff in measure 115 has a finger number of -6 and a *poco f* dynamic marking. The piece concludes with a double bar line.

116

Musical score for measures 116-118. The score is in 4/4 time and consists of four staves. Measure 116 features a treble clef with a key signature of one sharp (F#) and a 4/4 time signature. It includes a V-shaped fingering above a note with a finger number of -37 and a *pp espr.* dynamic marking. Measure 117 features a slur over a triplet of eighth notes with a finger number of -23, and a *p* dynamic marking. Measure 118 features a slur over a triplet of eighth notes with a finger number of -37, and a *pp* dynamic marking. The piece concludes with a double bar line.

119

Musical score for measures 119-121. The score is in 3/4 time and consists of four staves. Measure 119 features a treble clef with a key signature of one sharp (F#) and a 3/4 time signature. It includes a V-shaped fingering above a note with a finger number of -35 and a finger number of -49. Measure 120 features a V-shaped fingering above a note with a finger number of -47 and a finger number of -33. Measure 121 features a V-shaped fingering above a note with a finger number of -29 and a finger number of -43. The piece concludes with a double bar line.

124

Musical score for measures 124-128. The score is in 4/4 time and consists of four staves. Measure 124 starts with a treble clef, a key signature of one sharp (F#), and a dynamic marking of *p*. Fingerings are indicated as -18, -31, and -47. Measure 125 has a *sfp* dynamic. Measure 126 has a *sfp* dynamic and a 5/IV chord. Measure 127 has a 5/II chord and a -29 fingering. Measure 128 has a -27 fingering. The bottom staff has a -31 and -45 fingering.

129

Musical score for measures 129-132. The score is in 4/4 time and consists of four staves. Measure 129 has a *sfz p* dynamic and a -27 fingering. Measure 130 has a -25 fingering. Measure 131 has a -27 fingering. Measure 132 has a -12 and -10 fingering. The bottom staff has a -41 fingering. Dynamics include *sfz p* and *sfz p*.

133

Musical score for measures 133-136. The score is in 4/4 time and consists of four staves. Measure 133 has a -8 and -10 fingering. Measure 134 has a +6 and +4 fingering. Measure 135 has a +18 and +2 fingering. Measure 136 has a +16, +14, and +16 fingering. Dynamics include *sfp*, *sfz*, *p*, *f sfp*, *mp*, and *poco f*.

138

IV, V, III, I, II, III, V, III, +12, +10, +23, +37, +14, +27, +6, lower Db by one comma—match cello, sfz, 6.5

142

V, +6, +8, -4, +8, -8, +10, -6, +8, -20, c.l.b. → ord., come prima, II IV, sfz, 3

146

change gradually to harmonic, III, II, III, IV, -4, -18, -14, +1, sfz, p, III, II, III, IV, attaca, 3

Pythagoras Drawing (II)

Tempo ad libitum, vary from bar to bar as desired, swinging

ca. 66-99

The musical score is written for four instruments: Violin I (VI I), Violin II (VI II), Viola (Vla), and Violoncello (Vlc). It consists of three systems of four staves each. The first system covers measures 1-6, the second system covers measures 7-11, and the third system covers measures 12-15. The score includes various performance instructions such as *mezza voce, sostenuto*, *ff*, *martelé, full bow*, and *lead*. Fingerings and bowings are indicated throughout the piece. The tempo is *ad libitum* and the mood is *swinging*. The score is attributed to Villa Massimo, Roma, dated 5.7.2011 / 29.11.2011.