

Euler lattice with sruti regions from D represented by the simplest combinations of Helmholtz-Ellis accidentals a map of 23-limit harmonic space

for Bob Gilmore

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boxed numbers indicate division of the octave in 55 "commas"; top staff combines the Euler lattice with one additional 17-limit prime; second staff includes simpler combinations of higher primes up to 25; black noteheads indicate 46 peripheral ratios completing major/minor triads around the central 55-tone region (Euler Lattice staff) or ratios combining more than two accidentals (D staves)

dashed barlines: regions around distinct Euler lattice tones; barlines between staves: slightly larger gaps; ticks: tone regions within a few cents

cents values are calculated from pythagorean A taken as tuning reference (0 cents deviation on a tuning meter set to equal temperament); ratios are calculated from central pitch D; D^o comprises ratios taking their simplest form above D, D_o comprises the inversions (same intervals below D); ratios included: all 23-limit epimoric ratios up to the syntonic comma; all odd number multiple ratios from D; tuneable intervals between 1/1 and 2/1; simpler composite ratios from combinations of primes up to 13 (3, 5, 7, 11, 13)

System 0: Ratios include 1/1, 81/80, 65/64, 64/63, 55/54. Cents: +20, +25, +30.

System 1: Ratios include 513/512, 96/95, 2079/2048, 78/77, 77/76, 76/75, 70/69, 69/68, 66/65, 57/56, 56/55. Cents: +1, +16, +24, +20, +21, +21, +23, +24, +25, +29, +30.

System 2: Ratios include 45/44, 128/125, 6561/6400, 40/39, 416/405, 1053/1024, 36/35, 250/243, 49/48, 1701/1664, 52/51, 51/50, 50/49, 46/45, 39/38. Cents: +39, +41, +42, +44, +47, +36, +32, +33, +36, +42, +47.

System 3: Ratios include 33/32, 512/495, 28/27, 27/26, 20480/19683, 25/24, 2673/2560, 21/20, 4096/3969, 24/23, 22/21, 35/34, 34/33, 26/25, 23/22. Cents: +51, -43, -39, -33, -31, -28, -25, -17, +53, -50, -34, -31, -28, -25, -17.

System 4: Ratios include 256/243, 135/128, 18/17, 17/16, 16/15, 260/243, 15/14, 20/19, 96/91, 19/18, 81/76, 273/256, 171/160, 189/176. Cents: -12, -10, -3, +3, +10, +17, -13, -8, +8, +13, -13, -10, -8, -5, -3, +10, +17.

6 **7**

HE reduced I: 62, 320/297, Eb+27, +31, 27/25, Eb+37, 13/12, Eb+37, 64000/59049, +39, 243/224, +42, 88/81, 12/11, E-47, 2187/2000, -47, 35/32, 128/117, E-46, 3645/1664, E-44, 351/320, -42, 800/729, -41

HE reduced II: 69/64, +28

Euler Lattice: +31, E-47, -41

D°: 14/13, +26, +28, Eb+37, -51, -47

,D: 14/13, +26, -51

8 **9**

HE reduced I: 77, 11/10, E-37, 243/220, E-25, 72/65, 10/9, 9/8, +2

HE reduced II: 2048/1863, -38, 208/189, -36, 54/49, E-34, 256/231, -24, 5120/4617, -23, 285/256, -16, 64/57, -1, 91/81, Fb-0, 4617/4096, +5, 112/99, Fb+12

Euler Lattice: -20, 143/128, +2

D°: E-37, -20, -10, +2

,D: -20

10 **11**

HE reduced I: 93, 256/225, +22, 729/640, +23, 832/729, Fb+27, 585/512, E+29, 8/7, +29, 55/48, +34, 1024/891, +43, 144/125, +43, 280/243, Fb+43, 59049/51200, +45, 15/13, +46

HE reduced II: 92/81, +18, 108/95, +20, 18711/16384, +28, 147/128, Fb+38

Euler Lattice: +22, +23, +43, +45

D°: +29, +34, +46

,D: +29

12

HE reduced I: 108, 52/45, Fb+48, 204800/177147, -51, 81/70, +51, 125/108, -49, 297/256, E#-45, 64/55, -40, 7/6, -35, 243/208, E#-33, 2560/2187, -29, 75/64, -27

HE reduced II: 512/441, E#-44, 65536/56133, -34, 95/81, -26, 27/23, -24

Euler Lattice: -51, -49, -29, -27

D°: -35

,D: -40

13

14

122

HE reduced I: 189/160, 32/27, 1215/1024, 6/5, 65/54, 880/729

HE reduced II: 33/28, 45/38, 108/91, 19/16, 1024/855, 1539/1280, 77/64

Euler Lattice: 13/11

D°: F-13, -4

,D: -8, +14, +18, +14

15

136

HE reduced I: 40/33, 243/200, 1280/1053, 13312/10935, 39/32, 128/105, 8000/6561, 11/9

HE reduced II: 98/81, 63/52, 621/512

Euler Lattice: +35

D°: F+31, F+39, F+41, F+41, +45

,D: F+28, +30, +32, F+41, +45

16

17

147

HE reduced I: 27/22, 19683/16000, 315/256, 16/13, 100/81, 99/80, 56/45, 81/65, 5/4, 64/51

HE reduced II: 256/207, 704/567, 640/513, 1024/819, 304/243

Euler Lattice: F#-43, -37, -34, -19, -16, -16

D°: -47, -43, -37, -23, -16, -16, -9

,D: F#-42, F#-27, F#-15, 96/77, -20, -9

18

19

163

HE reduced I: 34/27, 512/405, 81/64, 80/63, 10240/8019, 32/25, 6561/5120, 104/81, 9/7

HE reduced II: 24/19, 91/72, 19/15, 14/11, 23/18

Euler Lattice: +4, +6, +12, +25, +27

D°: F#-1, +4, +6, +25, +27, +33

,D: -3, +2, Gb+3, F#+16, +25, F#+16, +25

20 **21**

178 165/128 128/99 162/125 35/27 135/52 13/10 25600/19683 125/96 2673/2048 72/55 21/16 256/195

HE reduced I +38 +47 +50 -48 -47 -45 G-41 G-31

HE reduced II 1323/1024 Gb+43 Gb+47 6656/5103 64/49 G-40 -36 -31 8192/6237

Euler Lattice +47 -47 -45

D° -48 -31

,D Gb+43 G-40

22 **23**

194 320/243 4/3 27/20 65/48 256/189 110/81

HE reduced I -25 +18 +23 +28

HE reduced II 95/72 243/184 2048/1539 171/128 128/95 13851/10240 693/512 G+23

Euler Lattice -25 +18

D° 169/128 G-21 -4 +18 G+23

,D -4

24 **25**

208 15/11 512/375 2187/1600 160/117 1664/1215 351/256 48/35 1000/729 11/8 243/176 112/81

HE reduced I +37 +39 +40 +45 +49 -43 -41

HE reduced II 49/36 567/416 G+35 G+42 G+44 G+45 16384/11907 +51

Euler Lattice +37 +39 G+45

D° +49

,D +45

26

222 177147/128000 18/13 81920/59049 25/18 891/640 7/5 729/520 1024/729 45/32 24/17

HE reduced I G#-39 G#-39 G#-29 G#-17 -19 -14 -12 -5

HE reduced II 32/23 88/63 80/57 128/91 38/27 G#-11 -10

Euler Lattice G#-39 -35 -33 -14 -12

D° -19 -12

,D G#-39 -30 -15 128/91 G#-11 -5

27 **28**

237 17/12 64/45 729/512 1040/729 10/7 1280/891 36/25 59049/40960 13/9 256000/177147

HE reduced I +1 +8 +10 +16 +26 +29 +31

HE reduced II 27/19 91/64 57/40 63/44 23/16

Euler Lattice +8 +10 +29 +31

D° +1 Ab+7 Ab+13 Ab+19 Ab+25 Ab+35 Ab+35

,D +8 +16

29

252 81/56 352/243 16/11 729/500 35/24 512/351 1215/832 117/80 3200/2187 375/256 22/15

HE reduced I +37 +40 +51 A-48 A-46 -44 -43 -41 A-39

HE reduced II 11907/8192 Ab+47 -49 -44 -43 -41 832/567 72/49 A-36

Euler Lattice +51 -43 -41

D° -49

,D Ab+47

30 **31** **32**

266 81/55 189/128 96/65 40/27 3/2 243/160

HE reduced I -32 -27 A-27 -22 +22

HE reduced II 1024/693 20480/13851 95/64 256/171 1539/1024 368/243 144/95 +22

Euler Lattice -26 -25 -18 -3 +3 +17 +18

D° -22

,D A-27 -22 256/169 A+17

33

280 195/128 32/21 55/36 4096/2673 192/125 19683/12800 20/13 208/135 54/35 125/81 99/64 256/165

HE reduced I +27 +32 +41 +43 +44 +49 A#-47 -42

HE reduced II 6237/4096 A+27 49/32 A+37 5103/3328 +38 A+46 -51 2048/1323 A#-45

Euler Lattice +26 A+36 +41 +43 -51

D° A+36 A#-47

,D +27 +44

34 **35**

296 14/9 81/52 10240/6561 25/16 8019/5120 63/40 128/81 405/256 27/17

HE reduced I A#-35 A#-25

HE reduced II A#-19 A#-7

Euler Lattice

D° A#-19

„D -37 Bb-3

36/23 11/7 30/19 144/91 19/12 192/121

36 **37**

311 51/32 8/5 130/81 45/28 160/99 81/50 13/8 512/315 32000/19683 44/27

HE reduced I +5 +12 +19 +33 +39 +43

HE reduced II 243/152 819/512 513/320 567/352 207/128

Euler Lattice Bb+11 Bb+23 Bb+39 Bb+39

D° +5 +16 77/48 Bb+39

„D +12 +43

38

327 18/11 6561/4000 105/64 64/39 10935/6656 1053/640 400/243 33/20

HE reduced I B-45 B-42 B-35

HE reduced II 1024/621 104/63 81/49 B-32

Euler Lattice B-45 -39

D° -49 B-44 -40 -39

„D -49 B-44

39 **40**

338 729/440 108/65 5/3 2048/1215 27/16 320/189

HE reduced I B-23 +2 +4 +10

HE reduced II 128/77 2560/1539 855/512 32/19 91/54 76/45 56/33

Euler Lattice -18 -18 Cb+2 Cb+14

D° -18 -18 +1 +4

„D -22 -21 -14 Cb+2 Cb+14 22/13 B+9

41 **42**

352

HE reduced I: 128/75 (+23), 2187/1280 (+25), 416/243 (Cb+29), 12/7 (+31), 55/32 (+36), 512/297 (+45), 216/125 (+45), 140/81 (+47), 177147/102400 (+48), 45/26 (+48)

HE reduced II: 46/27 (+20), 162/95 (+22), 56133/32768 (+30), 441/256 (Cb+40)

Euler Lattice: +23, +25, +45, +47

D°: +36

,D: +31

43

366

HE reduced I: 26/15 (-50), 102400/59049 (-49), 243/140 (-47), 125/72 (-47), 891/512 (-38), 96/55 (-33), 7/4 (-33), 1024/585 (-27), 729/416 (-25), 1280/729 (-25), 225/128 (-24), 95/54 (-22), 81/46 (-22)

HE reduced II: 256/147 (B#-42), 32768/18711 (-32)

Euler Lattice: -49, -47, -27, -25

D°: -33

,D: -50, -38, -33

44 **45** **46**

381

HE reduced I: 16/9 (-6), 9/5 (+16), 65/36 (+26), 440/243 (+26), 20/11 (+34)

HE reduced II: 99/56 (B#-16), 8192/4617 (B#-3), 162/91 (-2), 57/32 (-2), 512/285 (+12), 4617/2560 (+19), 231/128 (+20), 49/27 (+32), 189/104 (+32), 1863/1024 (+34)

Euler Lattice: -6, +16, +16

D°: +16

,D: -6, +6, +16, C+33

47

397

HE reduced I: 729/400 (+37), 640/351 (+38), 6656/3645 (+43), 117/64 (+43), 64/35 (+47), 4000/2187 (+47), 11/6 (+47), 81/44 (-45), 448/243 (-43), 59049/32000 (C#-41), 24/13 (C#-41), 50/27 (-35), 297/160 (C#-31), 128/69 (-32)

HE reduced II: 128/69 (-32)

Euler Lattice: +37, C#-41, -35

D°: +47, -30, 13/7 (-30)

,D: +43, +47, C#-41, -32, -30

48 **49**

412

HE reduced I: 28/15, C#-19, 15/8, 32/17, 17/9, 256/135, 243/128

HE reduced II: 352/189, C#-25, 320/171, C#-13, 512/273, 152/81, 36/19, 91/48, 19/10

Euler Lattice: -21, -17, -14, -12, +4, Db+5, +6, +8

D°: -21, -14, -7, -1, +4, Db+5, +9

,D: -21, -14, -7, -1, C#+1, +4, Db+5, +9

50 **51**

427

HE reduced I: 40/21, 5120/2573, 48/25, 19683/10240, 52/27, 27/14, 495/256, 64/33

HE reduced II: 21/11, Db+23, 23/12, 3969/2048, Db+45

Euler Lattice: Db+18, +24, +27, +29, +35, +40, Db+44, Db+45

D°: +14, 21/11, 44/23, +24, +27, +30, +35, +46, +48

,D: +14, Db+18, +21, +24, +27, +30, +35, Db+33, Db+45, 33/17, 68/35

442

HE reduced I: 243/125, 35/18, 2048/1053, 405/208, 39/20, 12800/6561, 125/64, 88/45

HE reduced II: Db+49, D-48, -46, -45, -43, 3328/1701, 96/49, D-38, -40

Euler Lattice: +49, -45, -43, 76/39, D-47, D-41, 45/23, D-38, 49/25, 100/51, 51/26, D-36, -37, -36

D°: Db+49, -46, -45, -43, 45/23, D-38, 49/25, 100/51, 51/26, D-36, -37, -36

,D: Db+49, -46, -45, -43, D-41, 45/23, D-38, 49/25, 100/51, 51/26, D-36, -37, -36

52 **53**

457

HE reduced I: 108/55, 63/32, D-29, 128/65, 160/81, 2/1

HE reduced II: -34, -29, 4096/2079, 95/48, 1024/513, -2, -5

Euler Lattice: -34, -29, -28, -23, -20, -5, -23, -2

D°: -34, -33, -29, D-29, 65/33, 136/69, 69/35, 75/38, 152/77, 77/39, D-24, -2, -2

,D: -34, -33, -29, D-29, 65/33, 136/69, 69/35, 75/38, 152/77, 77/39, D-24, -2, -2