“Well-Tempered” Extended JI Quartertone Tuning for Keyboard Instruments

based on a Harmonic Space subset defined by the prime partials 3, 5, 7, 11, 13, 17, 19, and 577

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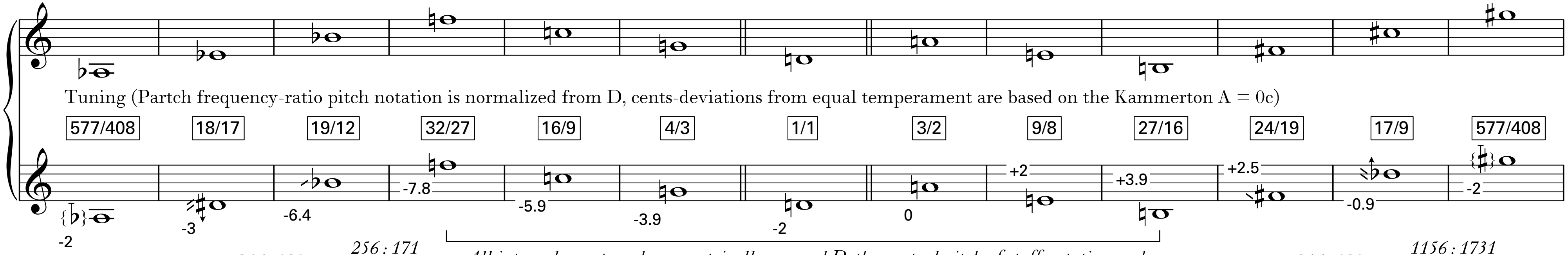
Key (tuning repeats in all octaves; either ignore inharmonicities or adjust slightly to reduce beats in the 2 : 3 ratios by tuning unisons between 2nd and 3rd partials)



*577 : 864 = 699c = 2 : 3 * (577 : 576) ca. -1/8 SC*

*216 : 323 = 696.6c = 2 : 3 * (324 : 323) ca. -1/4 SC*

*256 : 171 = 698.6c = 2 : 3 * (513 : 512) ca. -1/6 SC*

All intervals are tuned symmetrically around D, the central pitch of staff notation and traditional keyboard design. The six fifths between white keys are tuned Pythagorean in frequency proportion 2 : 3 = 702c. The six remaining fifths are each made slightly smaller, by approximately 1/6, 1/4 and 1/8 of a Syntonic Comma, producing a “circle” of fifths. The “chromatic” keys divide the Pythagorean wholetones into semitones 16 : 17 (C - Db and E - D#), 17 : 18 (Db - D and D# - D), 18 : 19 (A - Bb and G - F#) and 76 : 81 (Bb - B and F# - F).

*256 : 171 = 698.6c = 2 : 3 * (513 : 512) ca. -1/6 SC*

*216 : 323 = 696.6c = 2 : 3 * (324 : 323) ca. -1/4 SC*

*1156 : 1731 = 699c = 2 : 3 * (578 : 577) ca. -1/8 SC*

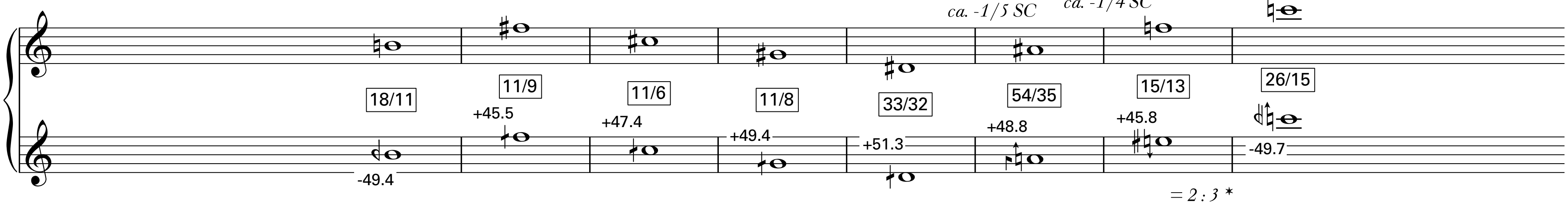
The ratio for G#/Ab is determined by dividing the wholetone G : A arithmetically in the frequency proportion 16 : 17 : 18 (obtaining an Ab) and harmonically in the frequency proportion 272 (=17·16) : 288 (=18·16) : 306 (=18·17) (obtaining a G#). The ratio G# : Ab, 288 : 289 (=17·17) is divided in the proportion 576 : 577 : 578. The intermediate pitch 577 produces a diminished fifth D - Ab of 600.003c, and a tritone Ab - D of 599.997c.

A second keyboard tuned as below augments the well-tempered tuning with quartertones.

*576 : 385 = 697.5c = 2 : 3 * (385 : 384) ca. -1/5 SC*

*175 : 117 = 697c = 2 : 3 * (351 : 350) ca. -1/4 SC*

*338 : 225 = 704.5c = 2 : 3 * (675 : 676) ca. +1/8 SC*

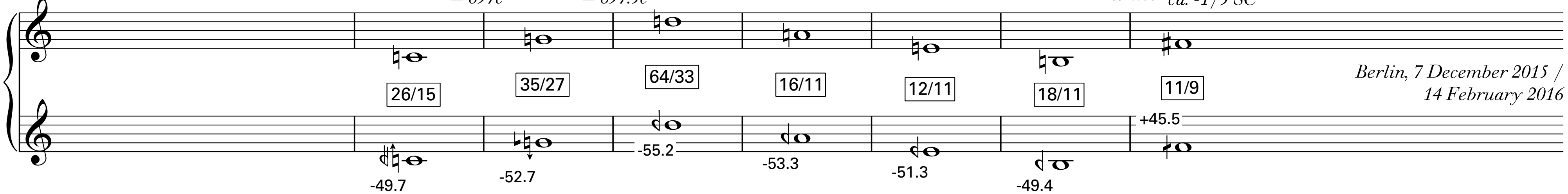


175 : 117 = 697c

576 : 385 = 697.5c

121 : 81 = 694.8c

*= 2 : 3 * (243 : 242) ca. -1/3 SC*



Berlin, 7 December 2015 / 14 February 2016